Optimal Irrigation on an Hourly Scale Using Model Predictive Control

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1 Introduction

It is estimated that 70% of Earth's freshwater is used for agriculture, and as global demand for water increases it becomes an even more valuable resource. Additionally, certain agricultural areas in the United States have experienced extended droughts due to changes in Earth's climate patterns which has also put a strain on water availability. Since maintaining proper soil moisture content is necessary to grow crops efficiently, optimal irrigation practices are critical.

We propose using Model Predictive Control (MPC) to minimize total water usage to irrigate crops. Using MPC controlled irrigation would provide benefits over standard feedback control because it would potentially allow for water dispersion to occur at optimal times during the day based on weather forecasting. For example, with a constraint on minimum soil moisture it may be optimal to irrigate more heavily when conditions are favorable and then abstain from irrigating for the next couple of hours. Moreover, while a MPC policy may choose to over-irrigate crops based on forecasting, the feedback system may abstain from irrigating crops because current soil moisture content is already at the desired level.

In this report we use an evapotranspiration model and soil water balance standardized by the Food and Agriculture Organization of the United Nations. Irrigation policies are then determined using Model Predictive Control theory to to minimize water usage for a corn field in Springfield, Illinois. Bounds on irrigation rate, water absorption rate, and total water held in the soil constrain our system. The cost function used in the MPC formulation is simply the integral of the water use rate for irrigation over the time period of predictive horizon.

2 System Dynamics

As described in (Vicente, 2011), (Brisson et al., 1992), (Walter et al., 2000), (Allen et al., 1998), (Aboitiz et al., 1986), and (FAO, 1999), the soil water balance equation is given as a discrete time difference equation.

$$SWC_{k+1} = SWC_k + IA_k - D_k - ETA_k + P_k \tag{1}$$

In Eq. 1, SWC is soil water content, IA is the irrigation amount, D is the drainage amount, ETA is the actual evapotranspiration, and P is the precipitation amount. All units are in mm. SWC is the state of interest in the system and IA is the control input. Each term is described in more detail in the appendix.

The drainage term is a piecewise linear term that overall makes our system non-linear, but piecewise affine. The ETA and P terms are measured variables that come from weather data such as temperature, day of year, cloud cover, etc. These equations are given in the appendix.

3 Model Predictive Control

The purpose of using a receding horizon controller for irrigation is to minimize the total amount of water used while maintaining the soil water content level above the stressed water threshold, SWC_{min}. By using MPC we will be able to incorporate weather predictions. Another benefit is that MPC implicitly utilizes feedback so that it is more robust to errors in weather predictions.

3.1 Cost Function

The overall goal is to minimize the total amount of water used to keep the field soil water content constraints satisfied. The cost function can be expressed formally as

$$\min J = \sum_{k=0}^{\infty} IA_k \tag{2}$$

When using receding horizon control this linear cost function changes to consider only the IA_k over the finite time horizon, N. Later in this report we will also augment the cost function to be able to handle losses due to spray irrigation losses.

3.2 Optimization

Model predictive control (receding horizon control) consists of solving the finite horizon problem at each discrete time step and implementing the first step of the resulting optimal control policy. The finite horizon problem that needs to be solved at each time step is

$$\begin{aligned} \min \quad & J = \sum_{k=0}^{N} IA_k \\ \text{subject to} \quad & \\ & SWC_k \geq SWC_{min} \\ & SWC_{k+1} = SWC_k + IA_k - D_k - ETA_k + P_k \\ & D_k = \begin{cases} 0 & SWC_k \leq FC \\ SWC_k - FC & SWC_k > FC \end{cases} \end{aligned}$$

However in this problem the constraints are non-linear due to the piecewise drainage term. In order to use traditional solution methods we must reformulate this problem to include linear constraints. It turns out that since the system dynamics are piecewise affine, this finite horizon problem can be formulated as a mixed-integer linear program by introducing some new variables. This procedure is discussed in more detail in (F. Borrelli, 2015).

3.3 Formulation as a MILP

The soil water balance equation as presented above is non-linear, but piecewise affine (PWA). This makes solving for optimal control more difficult than for linear systems. Casting the equation as a MILP will allow us to use branch and bound algorithms to solve the MPC finite horizon problem.

The soil water balance has two piecewise equations for system dynamics. We express these two cases as:

$$SWC_{k+1} = \begin{cases} FC + IA_k - ETA_k + P_k & SWC_k \geq FC \\ SWC_k + IA_k - ETA_k + P_k & SWC_k < FC \end{cases} \tag{3}$$

But we need to express our state equations as a single equation. By adding an event variable δ_k which acts as a logical operator we can express Eq 3 as a single state equation.

$$SWC_{k+1} = SWC_k(1 - \delta_k) + IA_k + FC\ \delta_k - ETA_k + P_k \tag{4}$$

where

$$\delta_k = \begin{cases} 1 & SWC_k \geq FC \\ 0 & SWC_k < FC \end{cases} \tag{5}$$

However, in Eq. 4 we introduce non-linear terms into the equation. We use a similar workaround and assign a new variable $z_k = SWC_k \delta_k$. Now, we reach a single, linear state equation that describes our system.

$$SWC_{k+1} = SWC_k - z_k + u_k + FC\ \delta_k - ETA_k + P_k \tag{6}$$

Given a linear cost function, Eq. 6 can be easily cast as a MILP. We have continuous variables SWC_k, z_k, and IA_k, and an integer variable δ_k. We apply the following linear constraints to complete the problem formulation and ensure the system behaves according to the system dynamics:

$$\begin{aligned} SWC_k &\geq SWC_{min} \\ SWC_k - FC &\leq M\delta_k \\ -(SWC_k - FC) &\leq -M(1 - \delta_k) \\ z_k &\leq M\ \delta_k \\ -z_k &\leq M\ \delta_k \\ z_k &\leq SWC_k + M(1 - \delta_k) \\ -z_k &\leq -SWC_k + M(1 - \delta_k) \\ \delta_k &\in \{0, 1\} \end{aligned}$$

Where M is arbitrarily large to satisfy our if-then statements for δ_k and z_k. Here, the first constraint states that we must keep the soil water content above a threshold. The next two equations describe the if-then case accompanying Eq. 5. Since $z_k = 0$ or $z_k = x_k$, similar logic is used to constrain z_k linearly in the next four constraint equations. The last constraint just states that δ_k is a binary integer.

3.4 Spray Irrigation Losses

Evaporation from the spray irrigation presents another avenue for moisture to escape during irrigation. The spray evaporation losses are not included in our soil model yet add to the water usage cost. In practice, whether or not we incorporate these losses into the optimization problem, these losses will have to be taken into account in order to maintain the proper SWC level. After the optimization problem is solved for irrigation amounts, an additional amount of irrigation is simply added based on the current weather induced losses.

However, we think it will be beneficial to incorporate these into the optimization problem. Spray losses can be modeled in the cost function as a terminal cost by including the control input as a state that we track. This terminal cost is given by $P_k x_f$ where P_k changes for each time k. This method is slightly complicated but is necessary because the losses are a function of weather and time of day. For brevity the full procedure for augmenting the cost function won't be given here. However the general idea is that each time the finite horizon problem needs to be solved with a new P_k matrix that contains the spray loss percentages at each time in the horizon. And the augmented state is such that part of the final state x_f contains each control input over the horizon and are multiplied by the corresponding spray loss percentages.

3.5 Implementation

To demonstrate the performance of the MPC control, three controllers were implemented. The baseline control was a simple derivative control that utilized the change in SWC in the last time step to implement a policy. This controller does not have any predictive capability and so we expect it to perform sub-optimally. Two MPC controllers were implemented: one without spray irrigation losses accounted for and one with them accounted for. This way we can see if utilizing this additional information reduces water usage as expected. We would have hoped to use a industry standard controller as a baseline but we were unable to find relevant information on current irrigation best practices (this would be looked into with more detail given more time).

All controllers and simulation were implemented in Matlab. The MPC problem was solved using a combination of HYSDEL and MPT (Herceg et al., 2013). HYSDEL is a Hybrid Systems Description Language that MPT can use to translate a PWA model into a linear system model. Essentially it is automating the process of formulating the problem as a MILP as discussed earlier. MPT also automates the solution of the finite horizon problem given an input cost function.

4 Results

Our model was first run without any control validate the dynamics. As shown in Fig. 1, our models for evapotranspiration and spray irrigation losses behave as expected for the given weather. The day-night cycle causes moisture loss to be at a minimum during the night while temperatures are low, solar radiation is nonexistent, and relative humidity is high. The model also responds to precipitation during the day as the lower temperature, cloud cover, and higher humidity reduce losses compared to a dry, sunny day.

With confidence in the model, the three controllers were applied for a case study on a single day near Springfield, Illinois. This case shown in Fig. 2, presents an interesting set of weather conditions that involves both a low starting SWC level and a precipitation event during the day. Simple derivative control shows the limitation of watering without predictive capabilities. An excessive amount of water is used to keep the soil water content above the threshold, and it actually irrigates during the same hour that it is raining!

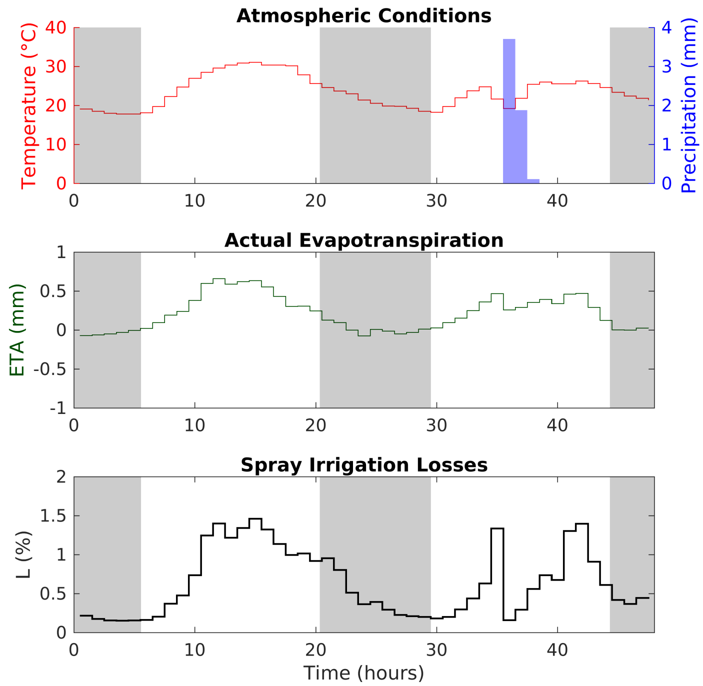

Figure 1: Actual Evapotranspiration and Spray Irrigation Losses

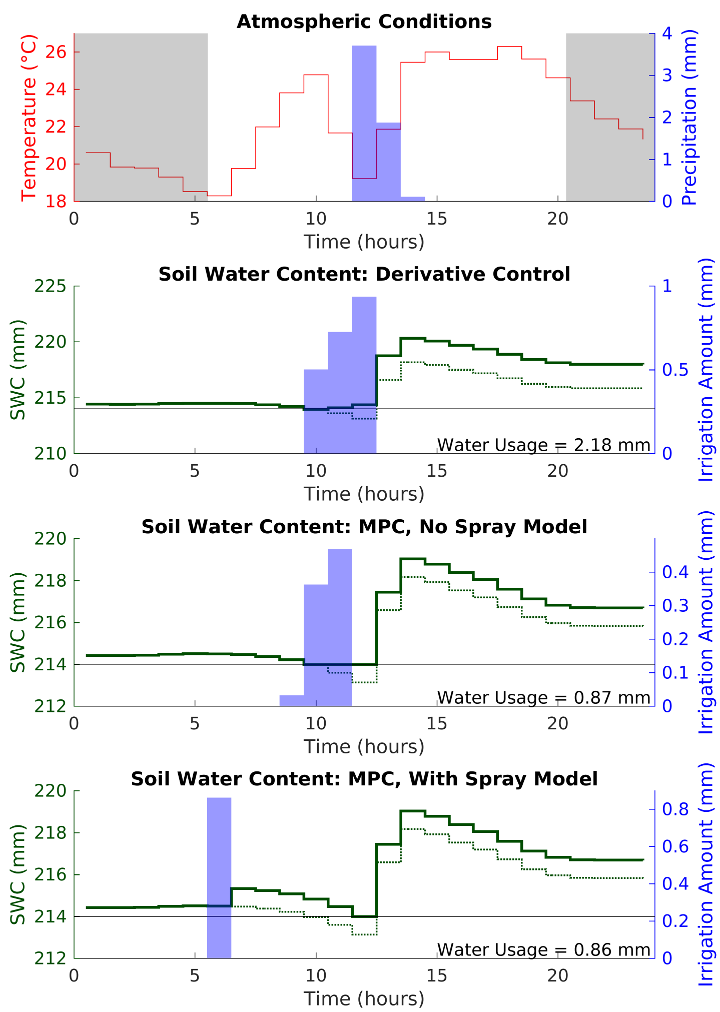

Figure 2: Comparisons between different controllers over the day of May 31, 2016. Both MPC controllers had a time horizon of $N = 6$ hours.

Meanwhile, MPC without the spray model adds just enough water to stay within our bounds. As a result, 1.31 fewer millimeters of irrigation is applied throughout the day. Obviously, knowing it will rain easily allows us to avoid excess water usage. The next step accounts for spray irrigation losses. During spray irrigation for crops such as corn, some water evaporates in the air. The amount of water lost depends on the weather. Once this is considered, our control understands it is most effective to irrigate during the morning when spray losses are at a minimum. Thus, all water is dispensed as early as possible while maintaining a soil water content within bounds before the precipitation comes. The incremental water saved with the spray model seems small but totals to a large volume across an entire farm.

The average size of a corn farm in Illinois is 358 acres, or $1.45(10^6)\ m^2$. When we ran our MPC controller over the course of the entire week of May 29 - June 4 the MPC with spray irrigation losses accounted for reduced the irrigation amount by 0.04 mm over the MPC without spray losses included. This is equivalent to saving $15,300$ gallons of water over the course of the week.

5 Conclusions

This report presents two different irrigation controllers using MPC, one with and one without spray irrigation losses. These controllers allow for the water dispersion to occur at optimal times during the day based on weather forecasting. The control policy was implemented for a corn producing farm in Springfield, Illinois from May 29 to June 4, 2016. The system dynamics had a piecewise affine function that had to be accounted for by considering it as a hybrid system. This resulted in a finite horizon problem that was solved with HYSDEL as a Mixed Integer Linear Problem. Unfortunately, we could not find the state-of-art irrigation control currently practiced in the industry. Hence, the performance of the proposed controllers are compared with a simple derivative controller as the baseline comparison. Based on our results, the MPC with spray irrigation losses modeled could potentially save a lot of water, even over an MPC controller without utilizing spray irrigation models. In our case study we save approximately $15,300$ gallons of more water over the course of the week for an average sized Illinois farm. The results corroborate that the MPC with spray irrigation losses outperforms other control policies mentioned in this paper and would be overall extremely beneficial to implement on real farms to reduce water usage.

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A Appendix

A.1 System Dynamics Relevant Equations

Drainage (D_k) The drainage is a piecewise function.

$$D_k = \begin{cases} 0 & SWC_k \leq FC \\ SWC_k - FC & SWC_k > FC \end{cases} \tag{7}$$

FC is the field capacity, or the maximum amount of water (in mm) that the soil in the crop root zone can hold. The drainage term simply describes that excess water above field capacity level is considered as drainage or runoff, meaning that no pooling occurs. The field capacity is determined by the type of soil. In Springfield, Illinois the soil is a silty loam, which can hold a volumetric percentage of $\theta_{max} = 31\%$. With a root depth for corn of 1 m, $FC_{corn} = 310$ mm.

Actual Evapotranspiration (ETA_k) The actual transpiration equation has been standardized by the Food and Agriculture Organization of the United Nations (Allen et al., 1998) (FAO, 1999), and is given in Eq. 8.

$$ETA_k = K_s K_c ET_{0,k} \tag{8}$$

In Eq. 8, K_s is the soil water constant, K_c is the crop coefficient, and $ET_{0,k}$ is the reference evapotranspiration.

$$K_s = \begin{cases} 1 & SWC_k \geq SWC_{stress} \\ \frac{SWC_k - SWC_{wilt}}{RAW} & SWC_{wilt} \leq SWC_k < SWC_{stress} \\ 0 & SWC_{wilt} > SWC_k \end{cases}$$

SWC_{stress} is the soil water content level when the crop begins having a difficult time extracting water from the soil. This amount is computed based on the guidelines given by (FAO, 1999),

$$SWC_{stress} = SWC_{wilt} + pTAW$$

In this equation SWC_{wilt} is the soil water content level at which the crop would become permanently wilted, $p_{corn} = 0.52$, and TAW is the total available water where

$$TAW = FC - SWC_{wilt}$$

For corn, the wilting point is $SWC_{wilt,corn} = 110$ mm and is based on a root depth of 1 m. This means that $SWC_{stress,corn} = 214$ mm.

The crop coefficient K_c is also a piecewise function of time, but is not a function of SWC. For corn, empirical models give:

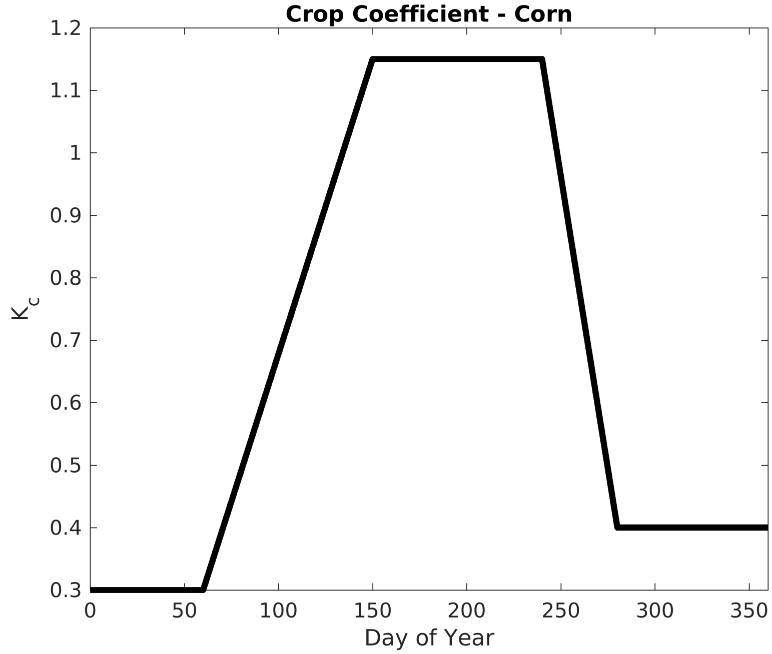

Figure 3: Crop Coefficient, K_c, for Corn

Reference Evapotranspiration ($ET_{0,k}$) The reference evapotranspiration equation is also standardized by the FAO.

$$ET_{0,k} = \frac{0.408\Delta(R_n - G) + \frac{\gamma C_n u(e_s - e_a)}{T+273}}{\Delta + \gamma(1 + C_d u)} \tag{9}$$

R_n is the net radiation (MJ/m^2h), G is the soil heat flux (MJ/m^2h), e_s is saturation vapor pressure (kPa), e_a is actual vapor pressure (kPa), Δ is the slope of the saturation vapor pressure-temperature relationship ($kPa/°C$), $\gamma = 0.0674$ is the psychrometric constant ($kPa/°C$), T is the hourly temperature ($°C$), u is the mean hourly wind speed (m/s), and C_n and C_d are constants defined as $C_n = 66$ and $C_d = 0.25$ (daytime) or $C_d = 1.7$ (nighttime). These values are computed using the equations below.

A.2 Reference Evapotranspiration Calculation

The following equations are from (FAO, 1999) and are used to compute values for Eq. 9.

$$
\begin{aligned}
\boldsymbol{G} &= \begin{cases} 0.1R_n & \text{daytime} \\ 0.5R_n & \text{nighttime} \end{cases} \\
\boldsymbol{e_s} &= \frac{1}{2}(e_{0,max} - e_{0,min}) \\
e_{0,max} &= 0.6108e^{\frac{17.27T_{max}}{T_{max}+237.3}} \\
e_{0,min} &= 0.6108e^{\frac{17.27T_{min}}{T_{min}+237.3}} \\
\boldsymbol{e_a} &= 0.6108e^{\frac{17.27T_{dew}}{T_{dew}+237.3}} \\
\boldsymbol{\Delta} &= \frac{4098(0.6108e^{\frac{17.27T}{T+237.3}})}{(T+237.3)^2} \\
\boldsymbol{R_n} &= R_{ns} - R_{nl} \\
R_{nl} &= \frac{1}{24}\sigma(\frac{T^4_{max,k} + T^4_{min,k}}{2})(0.34 - 0.14\sqrt{e_a})(1.35\frac{R_s}{R_{so}} - 0.35) \\
R_{s0} &= \frac{3}{4}R_a \\
R_{ns} &= (1-a)R_s \\
R_s &= (\frac{1}{4} + \frac{1}{2}\frac{n}{60})R_a \\
R_a &= 18.8d_r((w_2 - w_1)\sin\phi\sin\delta + \cos\phi\cos\delta(\sin w_2 - \sin w_1) \\
\delta &= 0.409\sin(\frac{2\pi J}{365} - 1.39) \\
w_1 &= w - \frac{\pi}{24} \\
w_2 &= w + \frac{\pi}{24} \\
w &= \frac{\pi}{12}(t_k + S_c - 12) \\
S_c &= 0.1645\sin(2b) - 0.1255\cos b - 0.025\sin b \\
b &= \frac{2\pi(J-81)}{364} \\
d_r &= 1 + 0.033\cos(\frac{2\pi J}{365})
\end{aligned}
$$

The mean saturation vapor pressure e_s is determined from air temperatures T_{max} and T_{min}, and actual vapor pressure e_a can be determined by the dew point temperature T_{dew}. R_a is the extraterrestrial radiation, and is a function of latitude, date, and time of day. R_s is the shortwave radiation, and is the amount of radiation that actually reaches the surface of Earth. R_{so} is the clear-sky solar radiation. R_{ns} is the net solar radiation and is based on a which is the albedo of the particular crop ($a = 0.2$ for corn). R_{nl} is the net long wave radiation that is lost by the Earth back into space. d_r is the Earth-Sun inverse relative distance, and δ is the solar declination. J is the day of year $(0 - 365)$, w is the solar time at midpoint of the hour period, w_1/w_2 are the solar time angle at the beginning/end of the hour (rad). S_c is the seasonal correction for solar time. $\sigma = 4.903(10^{-9})$ (MJ/K^4m^2day) is the Stefan-Boltzmann constant, n is the number of minutes of daylight in the hour, ϕ (rad) is the latitude. Several other constants have also been used in these equations but not defined. For more information see (FAO, 1999).