Application of a Genetic Algorithm to the Optimization of Walker Delta Constellations

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***Abstract*—Since traditional satellites are too large and expensive, recent advances in spaceflight have specifically focused on satellite constellations, or distributed space systems (DSS). Coordination between a team of smaller cubesats, for example, can enable the distribution of workload, offer greater mission flexibility and safety, and significantly reduce overall cost. In this report, the Walker Delta Constellation (WDC) is considered, and a method for satellite constellation optimization is implemented based on genetic algorithms (GA). The GA evaluates the overall performance, or fitness, of the WDC by evaluating the percent coverage of the Earth. The method is applied to several example problems in order to demonstrate the effects of constellation design parameters and orbital configurations on Earth coverage performance.**

***Index Terms*—Optimization, Genetic Algorithm, Walker Delta Constellation, Orbital Mechanics**

I. Introduction

UNTIL recently, most space missions were designed to be completed by a single spacecraft. These traditional satellites are large, expensive, and often limit mission objectives; therefore, recent advances in spaceflight have focused on utilizing satellite constellations, or distributed space systems (DSS). DSS use a series of small satellites in different orbits to achieve the mission objectives. Coordination between a team of smaller satellites can enable the distribution of workload, offer greater mission flexibility and safety, and significantly reduce overall cost. In particular, DSS can provide the following advantages: 1) wide global coverage and 2) frequent regional access[1].

This project focuses on optimization of DSS to maintain continuous global coverage for Earth observation and monitoring. The Walker Delta Constellation (WDC) is the subject of this study, which aims to determine optimal orbital parameters and constellation configuration that results in maximum, continuous ground coverage of the Earth.

II. WALKER DELTA CONSTELLATION

The design of satellite constellations is extremely difficult, partly because there are an infinite number of options for choosing the six Keplerian orbital elements that characterize an orbit. The six Keplerian orbital elements are: semi-major axis, a, eccentricity, e, inclination, i, argument of perigee, ω, right ascension of the ascending node, Ω, and mean anomaly, M. In some cases, the mean anomaly can instead be represented by the true anomaly, ν. Figure 1 provides a visual representation of these six parameters.

One important pioneer of satellite constellation design was John Walker in the 1970s. During his time at the British Royal Aircraft Establishment, Walker applied a geometrical approach towards constellation design to better understand the feasibility of multiple satellites for communication purposes[2]. The result was the notable WDC, which is a symmetric constellation consisting of circular orbits at a common altitude and inclination.

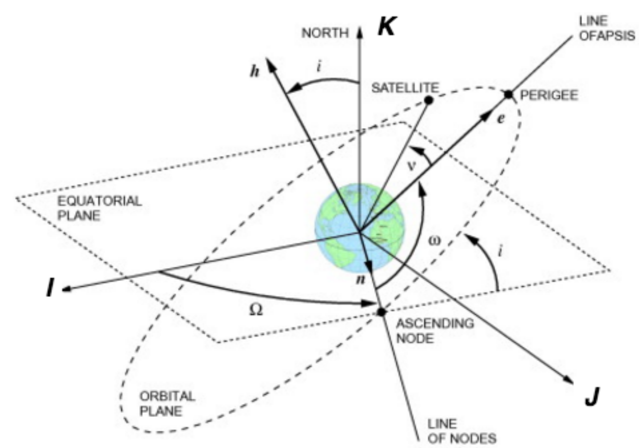

Fig. 1: Representation of the Keplerian orbital elements.

The WDC is completely characterized by five parameters: the total number of satellites, T, the number of orbital planes, P, a phasing parameter between satellites on different orbital planes, F, the orbital altitude, h, and the orbital inclination, i[3]. WDCs are, by definition, symmetric so there must be S uniformly distributed satellites per plane

$$S = \frac{T}{P}. \tag{1}$$

Note that S must be an integer, so T must be a multiple of P. The symmetric arrangement of the satellites in their respective planes is given by a constant phase difference, which is defined by a pattern angle

$$\gamma = \frac{2\pi}{T}. \tag{2}$$

The phase difference between satellites on *different* orbital planes is defined by $F\gamma$, where $F \in [0, P\text{-}1]$. Furthermore, the orbital planes are configured such that the symmetry is achieved by uniformly distributing the ascending nodes according to

$$\Omega_k = \frac{2\pi k}{P}, \tag{3}$$

where $k \in [1, P]$. Lastly, the mean anomaly for any arbitrary number of satellites in the orbit is given by

$$M_{k,l} = lF\gamma + kS\gamma, \tag{4}$$

where $l \in [1, S]$. Ω and M are defined in the interval $[0, 2\pi]$.

III. PROBLEM FORMULATION

For a single satellite, the ground coverage can be computed from this geometric equation

$$\sin\eta = \frac{R_E}{h + R_E}\cos\epsilon = \cos(\lambda + \epsilon), \tag{5}$$

where η is the satellite viewing angle, R_E is the radius of the Earth, h is the altitude, λ is the Earth central angle, and ϵ is the elevation angle[4]. These parameters are visually represented in Figure 2. In the figure, SSP denotes the sub-satellite point, which is the point on the Earth that sees the satellite directly above (i.e., with an elevation of 90^o). The fraction of the Earth surface seen by the satellite at any given time is called the access area. It is easy to see that the size of the access area directly depends on the altitude and the sensor field of view, which is fixed at 30^o. The elevation angle was also fixed at 5^o. As long as the spacecraft elevation angle is greater than zero, the satellite will be visible to a observer located on the surface of the Earth; but, a minimum elevation angle is specified to account for buildings or mountains that may be in the line of sight between the observer and the spacecraft.

The following formulation provides a simplified analytical approach to calculating the access area, or equivalently the coverage area, for a *single* satellite:

$$A_{COVERAGE} = A_C = 2\pi R_E^2(1 - cos\lambda). \tag{6}$$

This equation multiplied by the total number of satellites in the constellation results in the total surface area that is covered at any given time by the satellites. This equation represents the most basic approach for quantifying the percent coverage, and it can be further complicated by accounting for Earth's oblateness, overlapping coverage from multiple satellites, or specifying sensor/instrument characteristics, for example[5]. For the scope of this project, this simple formulation is sufficient.

The total Earth coverage area, in percent, for specific Walker Delta Constellation parameters can be computed as:

$$A_C = \frac{P \cdot S}{2} \cdot \left[1 - \cos\left(\arccos\left(\frac{R_E\cos\epsilon}{h + R_E}\right) - \epsilon\right)\right] \times 100. \tag{7}$$

Hence, the optimization is devised as:

$$\underset{P,h,S,i}{\text{argmin}} \quad \left\{\frac{P \cdot S}{2}\left[1 - \cos\left(\arccos\left(\frac{R_E\cos\epsilon}{h + R_E}\right) - \epsilon\right)\right] - 1\right\}^2,$$

subject to

$$\begin{aligned} &1 \leq P, S \leq 10 \\ &300 \leq h \leq 1000\,\text{km} \\ &45^o \leq i \leq 90^o \\ &\epsilon = 5^o = 0.873\,\text{rad} \\ &R_E = 6378\,\text{km}, \end{aligned}$$

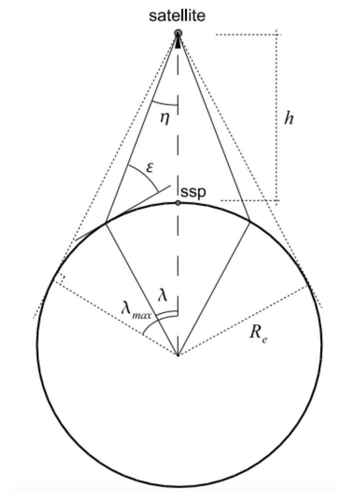

Fig. 2: Satellite viewing geometry.

where the cost is minimized at 100% of Earth coverage. The constraints are chosen such that the satellites are in Low Earth Orbit (LEO), all geographical latitudes can be observed, and the total number of satellites do not exceed 100, which could be a cost constraint specified by the customer.

In particular, atmospheric drag on the satellite is a major concern in LEO, so a minimum altitude of 300 km ensures that the orbit does not decay (i.e., spiral towards the center of the Earth). On the other hand, a maximum altitude of 1000 km provides a safety margin for the satellite such that it does not encounter the Earth's Van Allen Radiation Belts, which could interfere with communications and potentially damage satellite instruments. The inclination impacts how coverage patterns are formed, as well as coverage as a function of the geographical latitude. For example, a satellite at an inclination of 60^o can observe, at most, $\pm$ 60^o of latitude about the Earth's equator. Any portion of the Earth outside of this range, such as the North and South poles, will not be visible to the satellite. Lastly, the maximum number of satellites in the constellation is a practical constraint, since the cost (i.e., for launch, operations, maintenance, etc.) increases exponentially with the number of satellites.

IV. GENETIC ALGORITHM

The optimal solution to the aforementioned problem can be evaluated using various optimization algorithms, such as gradient descent and conjugate descent methods. Genetic algorithms (GA) have also been considered in the past, since they are compatible with any cost function and convergence is independent of initial conditions[6]. The GA is one of the population methods that optimizes collection of design points. The large number of design points (i.e., individuals or chromosomes in GA) spread over the whole feasible domain facilitates the search of the global minimum. As the name implies, the algorithm was inspired from biological evolution. The individuals undergo selection, crossover, and mutation at every generation to diversify their properties to explore the domain.

A. Initialization

Genetic Algorithm requires randomly generated chromosomes for the initial populations. Typically chromosomes are real-valued and binary strings are used for DNA and other applications. This paper implements 5 different real and integer values for each chromosomes, which are: 1) the number of orbital planes, P, 2) altitude, h, 3) the number of satellites per orbital plane, S, 4) inclination angle, i, and 5) cost. The values of each properties excluding cost are randomly generated within constraints. Specifically, the number of orbital planes and satellites per orbital plane are a subset of integers (i.e., $P, S \in \mathbb{Z}$). The altitude and inclination angles are a subset of real numbers ($h, i \in \mathbb{R}$). The parameters are a subset of feasible domain (i.e., abiding constraints).

B. Selection

The selection is the process of choosing parents for the next generation of chromosomes. The *m* parental pairs generates *m* children of the next generation. There are three different approaches for biasing the selection towards the fittest. The truncation selection (i.e., the algorithm implement in this paper) is uniform selection of the best (i.e., fittest) *k* chromosomes in the population. The truncation (i.e., elitism) selection rate was chosen as 30% of the total population in this paper. The tournament selection is randomly choosing *k* parents in the population. The roulette wheel selection (a.k.a. fitness proportionate selection) chooses each parent with a probability proportional to its performance relative to the population.

C. Crossover

After choosing the parents through selection, crossover combines the parents' chromosomes to form children. There are three different crossover schemes: single point and two points crossovers, and uniform crossover. In single point crossover, a child gets chromosomes from parent A up to the randomly (i.e., uniformly random) chosen point and receives remaining chromosomes from parent B, and vice-versa for the other child. Similarly, two crossover points uses two randomly chosen points. The uniform crossover gives equal chance for each chromosome to be inherited from either parents. Hence, it is likely that uniform crossover will generate alternatively receiving chromosomes from each parent. This paper implements single point crossover.

D. Mutation

In order to explore unprecedented traits, chromosomes are mutated such that they have new traits (i.e., evolution). Mutation occurs for all children after crossover at the rate inversely proportional to the number of bits in a child which yields an average of 1 mutation per child chromosome. The mutation is done by flipping bit for binary-valued and adding zero-mean Gaussian noise for real-valued chromosomes. The mutation in this paper was done by adding zero-mean Gaussian with the standard deviation of 50% of its trait.

The genetic algorithm implemented in this paper is delineated in Algorithm 1.

Algorithm 1 Genetic Algorithm for the Walker Delta Constellation Optimization

Inputs:
size α of population = 10000,
rate β of elitism (selection rate) = 0.3,
rate γ of mutation = 0.5,
number δ of iterations = 1000

Outputs:
number of orbital planes: P,
altitude [km]: h,
number of satellites per plane: S
inclination angle [^{o}]: i

procedure GA
  Initialization
  generate α feasible solutions randomly;
  save them in the population Pop;
  for $i = 1$ `to` δ **do**
    Truncation (elitism based selection)
    number of elitism: $ne = \alpha \cdot \beta$;
    select the best ne solutions in Pop and save them in Pop_1;
    Crossover
    number of crossover: $nc = \frac{(\alpha - ne)}{2}$;
    for $j = 1$ `to` nc **do**
      randomly select two solutions X_A and X_B from Pop;
      generate X_C and X_D by one-point crossover to X_A and X_B;
      save X_C and X_D to Pop_2;
    end for
    Mutation
    for $j = 1$ `to` nc **do**
      select a solution from Pop_2;
      mutate each bit of X_j under the rate γ and generate a new solution X'_j;
      if X'_j is infeasible **then**
        update X'_j with the closest value within the feasible set;
      end if
      update X_j with X'_j in Pop_2;
    end for
    Update and Sort
    update $Pop = Pop_1 + Pop_2$;
    sort Pop with ascending cost;
  end for
  return Pop ▷ The first row in Pop is the optimal solution
end procedure

V. RESULTS

The four Walker Delta Constellation parameters are optimized with the proposed GA. The 15 best results are tabulated in Table I. The results are converged to 30 satellites (T), 5 orbital planes (P), 6 satellites per orbital plane (S), altitude (h) of 693.53 km, and inclination angle (i) of 68.18^o. Note that either P and S could be 5 or 6, because the product of them is the number of satellites which optimizes to be 30.

Rank	T	P	h	S	i
1	**30**	**5**	**693.53**	**6**	**68.18**
2	30	5	693.53	6	68.26
3	30	6	693.53	5	68.28
4	30	6	693.53	5	68.21
5	30	5	693.53	6	68.24
6	30	5	693.53	6	68.23
7	30	5	693.53	6	68.20
8	30	5	693.53	6	68.21
9	30	6	693.53	5	68.19
10	30	6	693.53	5	68.26
11	30	6	693.53	5	68.24
12	30	6	693.53	5	68.27
13	30	5	693.53	6	68.25
14	30	6	693.53	5	68.21
15	30	5	693.53	6	68.25

Table I: The properties of the best 15 chromosomes in the population after the last generation.

The cost and coverage area contour plots are depicted in Figures 3 and 4 respectively. In Figure 3, the cost function shows convex trend as either the number of satellites and the altitude increases. The trend agrees with the expectation that an insufficient number of satellites will not cover the whole Earth and excess satellites will lead to overlapping, redundant coverage. Similarly, the satellites at very low altitudes will not able to cover the whole Earth due to the limited field of view, and the satellites in high altitude can cover larger area, which leads to excess coverage and high cost (i.e., the launch cost associated with placing a satellite at a high altitude).

In Figure 4, the coverage area increases proportionally with the altitude and number of satellites. However, the coverage area cannot reach 100% by solely increasing the altitude or the number of satellites. The two parameters have to be increased sufficiently to reach the target. The results shows that the optimal number of satellites and the altitudes are approximately 30 and 700 km, respectively.

The main objective of this paper is to optimize coverage with respect to the altitude and the number of satellites. Hence, cost and percentage coverage area are represented with respect to each parameter in Figures 5 and 6. The cost curve exhibits convex trend while coverage area increases with each parameter as explained before. In Figure 5, the cost is minimized and the percentage coverage area reaches 100% at 30. Similarly in Figure 6, the cost is minimized and the percentage coverage area reaches 100% approximately at 700 km.

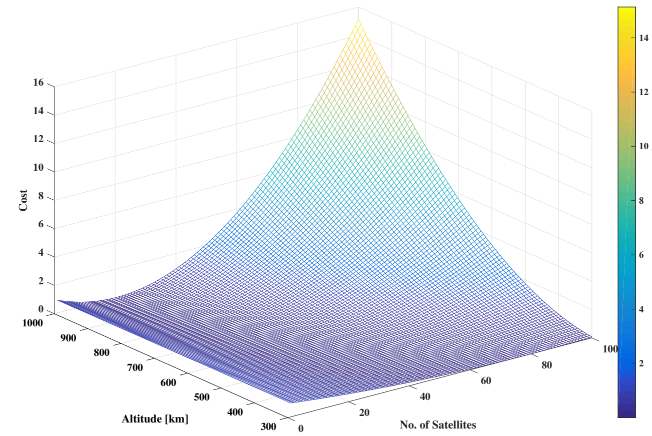

Fig. 3: The cost contour plot of altitude and the number of satellites.

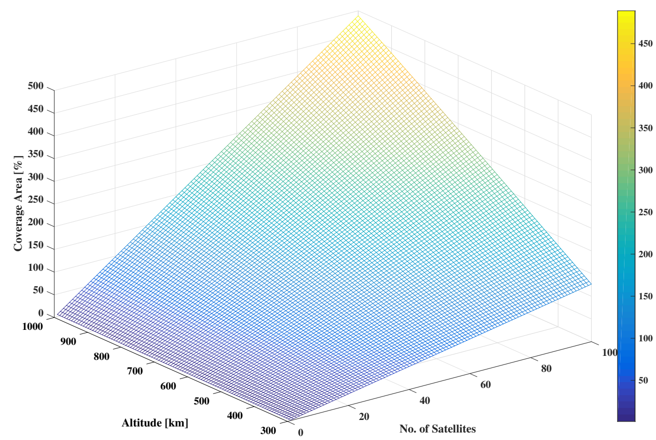

Fig. 4: The coverage area contour plot of altitude and the number of satellites.

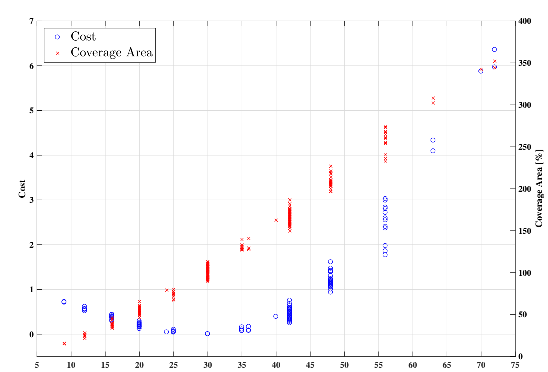

Fig. 5: The cost and coverage area plots of the number of satellites.

The convergence trend of the Genetic Algorithm is

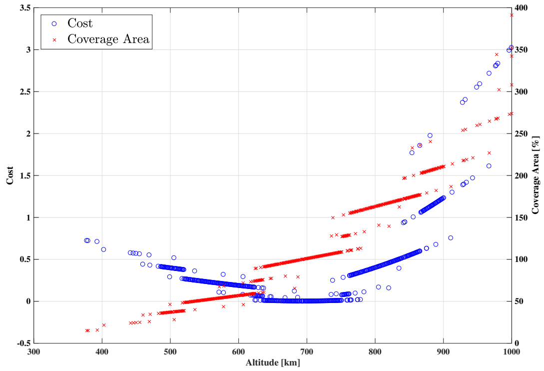

Fig. 6: The cost and coverage area plots of the altitude.

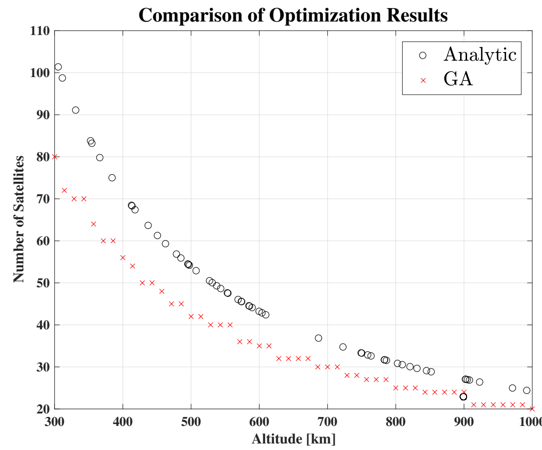

Fig. 8: The comparison plot of analytic and GA optimizations.

represented in Figure 7. The plots show that approximately 8500 out of 10^4 chromosomes converged to the optimized results after 1000 generations. The other remaining chromosomes are deviates from the optimal values with increasing rank as expected.

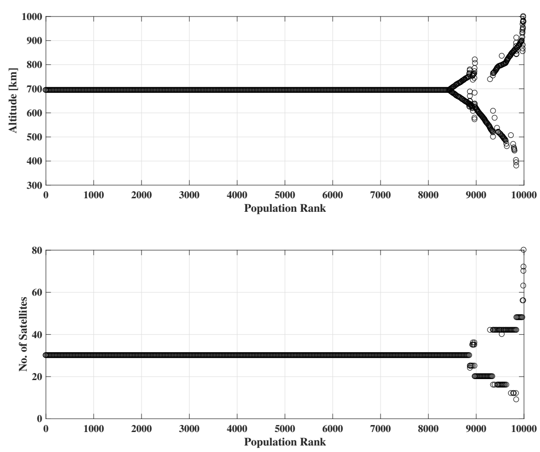

Fig. 7: The altitude and the number of satellites of the last generation chromosomes.

In Figure 8, the benchmarking was performed on the relationship between the altitude and the number of satellites. Indeed, the GA shows the same trend as the analytic equation implying that the algorithm is functioning as expected. Note that the GA curve is shifted left compared to the analytic curve. This effect is due to the consideration of multiple inclination angles in GA, whereas the analytic equation is independent of inclination.

The approximate coverage of the Walker Delta Constellation over the 2D Earth map is shown in Figure 9. Note that the circles almost cover up the whole map. However, due to the time constraint and the lack of knowledge of authors in determining shape of the circle projected near the poles, the circles near the poles are not represented accurately. The coverage area should get more wider as it approaches the north and south poles. Therefore the actual and accurate plot will cover the whole 2D plot for the proposed optimal solution.

VI. Conclusion and Future Work

In this paper, a GA was implemented for solving the optimization problem regarding Earth observation and monitoring with WDCs. The problem was formulated such that the minimum number of satellites can obtain continuous, global coverage of the Earth. The optimization was subject to orbital constraints, governed by practical considerations such as the effect of the space environment on teh spacecraft, launch and maintenance costs, and regional access. Numerical results yielded an optimal WDC configuration that has 30 total satellites, distributed over 5 planes, at an altitude of 693.53 km, and an inclination of 68.18^o. The numerical results were shown to be consistent with analytic models, and the optimized result to be practically feasible.

Future research opportunities include improving the GA to be more robust and encourage exploration by changing the selection mechanism, mutation rate, and update to two-point cross over. These changes could potentially lead to a better optimal result.

VII. Contributions

Jin Woo and Harsh contributed equally for this project.

References

[1] Bruccoleri, Christian. “FLOWER CONSTELLATION OPTIMIZATION AND IMPLEMENTATION.” Texas A&M University. December 2007.

[2] J. Walker, Some Circular Orbit Patterns Providing Continuous Whole Earth Coverage,Journal of the British Interplanetary Society, Vol. 24, 1971, pp. 369384.

[3] J. Walker, Circular Orbit Patterns Providing Continuous Whole Earth Coverage, Royal Aircraft Establishment Technical Report #70211, November 1970.

[4] Adams, W., and L. Rider. ”Circular Polar Constellations Providing Continuous Single or Multiple Coverage Above a

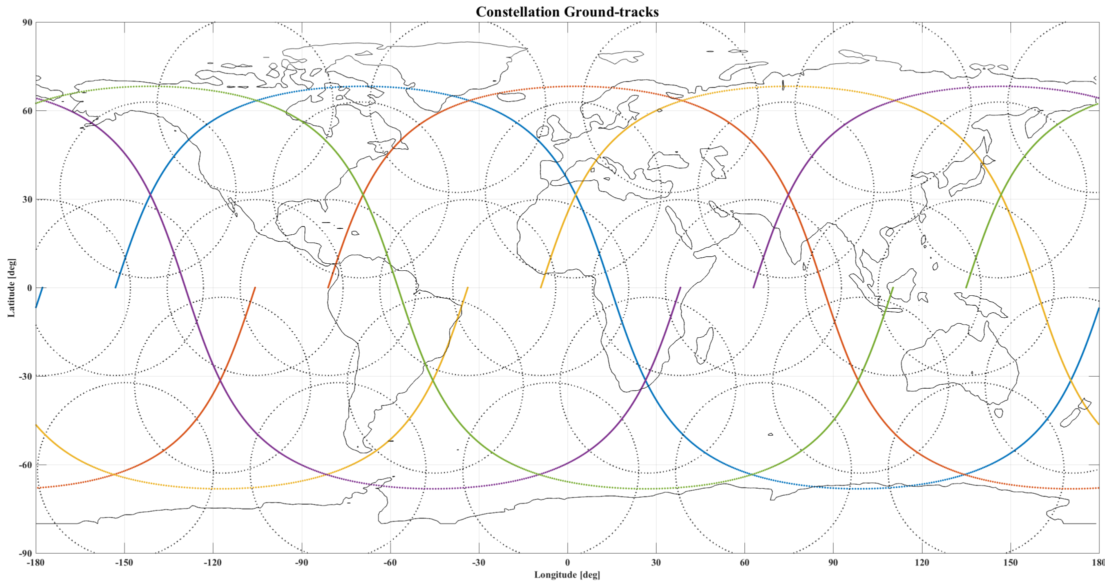

Fig. 9: The coverage plot of the optimized results.

Specified Latitude." The Journal of the Astronautical Sciences, Vol XXXV, No. 2, pp. 155-192, Apr-Jun 1987.

[5] Lang, T. "Symmetric Circular Orbit Satellite Constellations for Continuous Global Coverage." AAS paper 87-499, AAS/AIAA Astrodynamics Specialist Conference, Kalispell, Montana, Aug 10-13, 1987.

[6] Whittecar, William and Ferringer, Matthew. "GLOBAL COVERAGE CONSTELLATION DESIGN EXPLORATION USING EVOLUTIONARY ALGORITHMS." AIAA Space Forum. August 2014.

[7] Ely, Todd A. et al. "Satellite Constellation Design for Zonal Coverage Using Genetic Algorithms." Journal of the Astronautical Sciences, Vol 47, No. 3. July - December 1999, pp 207 - 228.

[8] Walker, J. "Continuous Whole Earth Coverage by Circular Orbit Satellite Patterns." Royal Aircraft Establishment Technical Report #77044, March 1977.

[9] Rider, L. "Analytic Design of Satellite Constellations for Zonal Earth Coverage Using Inclined Circular Orbits." The Journal of the Astronautical Sciences, Vol.34, No. 1, Jan-Mar 1986, pp. 31-64.15.

[10] Larson, Wiley J. and Wertz, James R. *Satellite Mission Analysis and Design*. Microcosm Press, El Segundo, CA. 3rd edition. 1999.

[11] R. Luders, Satellite Networks for Continuous Zonal Coverage,American Rocket Society Journal,Vol. 31, 1961, pp. 179184.